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## DYNAMICAL EVOLUTION AND STATISTICS OF DAMOCLOIDS. 1

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The orbital evolution of 144 Damocloids and their clones (12 for each real object, a total of 1872 test particles) is modeled for 100 million years in the future. It is shown that the population of Damocloids retains high orbital inclinations during the integration time of 100 million years. By their dynamic life time, Damocloids can be divided into two subpopulations, with a short and long time of life. The first includes 85.75% of the objects and has an average life span of  $2.68 \pm 0.04$  million years. The subsistence population with a long life span is 14.25%, with an average lifetime of  $126 \pm 6$  million years. The main parameter by which the division can be made without prior simulations is perihelion distance: We found 5 interesting objects that have a long time of life, but in parameter space they occupy positions among objects with a short life span. Taking into account the bimodal distribution of Damocloids in a dynamic life span, it is proposed to clarify the definition of Damocloids by perihelion distance  $q$  (more or less than 9 AU). We propose to name these long-lived and short-lived subpopulations as trans-Saturnian and pre-Saturnian Damocloids.

**Keywords:** damocloids – comets – evolution – statistics

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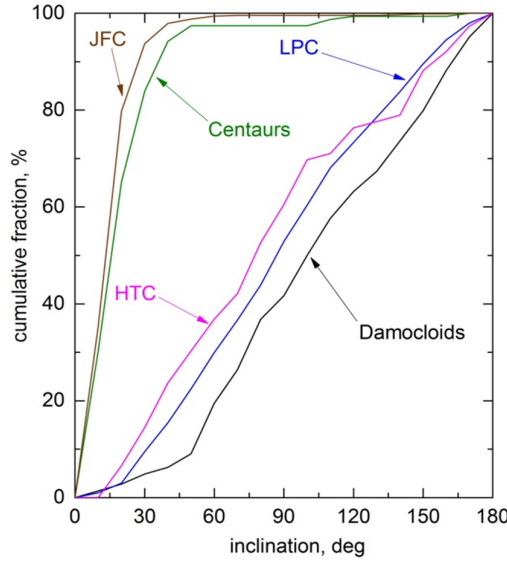
## 1. INTRODUCTION

Damocloids are selected among minor bodies as objects satisfying the two criteria: they show no cometary activity (and thus are classified as asteroids), and they have the Tisserand parameter  $T_J \leq 2$  [1]. 144 Damocloids have been discovered by the time our simulation began according to JPL HORIZONS [2]. The Damocloids orbits are mostly high inclined to the ecliptic plane (from  $1.7^\circ$  to  $175.2^\circ$ ). 84 from 144 Damocloids move on retrograde orbits. Their orbital eccentricities are in a range from 0.2523 to 0.9983. Damocloids' perihelia are from 0.205AU up to 23.797AU. Damocloids' aphelia are from 4.57 AU up to 3176.53 AU.

Damocloids are believed to be Halley-type cometary nuclei originating in the Oort cloud that have lost their volatiles [1]. Some assigned Damocloids later showed cometary activity. The first Damocloid, (5335) Damocles, was discovered in 1991, and the first retrograde Damocloid, discovered in 1999, was named Dioretsa (20461) - the name is "asteroid" spelled vice versa. Damocloids' retrograde orbits provide one argument more in favor of their relationship with the Oort cloud and Halley-type comets. In the present study we focused on several objectives of the work: What is the dynamic lifetime of Damocloids' population and its dynamic link to the hypothetical Oort cloud and Halley-type comets [3]? For how long can their inclined orbits be preserved? How often the transitions from direct to retrograde motion or vice versa happens? What is the mechanism of such transitions? Can distant Damocloids move closer to the sun thus replenishing the population of Halley-type comets [4,5]? Can Damocloids replenish the near-Earth region?

## 2. CALCULATION METHOD

To simulate the dynamical evolution of Damocloids we applied the RMVS integration technique (Regularized Mixed Variable Symplectic method [6, 7]) from the SWIFTER package. We considered the presence of Lidov-Kozai mechanism in transition of Damocloids from prograde to retrograde motion. In celestial mechanics, the Lidov-Kozai mechanism, effect, or resonance is the periodic change in the ratio of the eccentricity and inclination of the orbit under the action of a massive body or bodies. The second manifestation of this effect is libration (oscillations around a stable value) of the argument of the pericenter. This effect was described in 1961 by M. L. Lidov when he studied the orbits of artificial and natural satellites of the planets [8], and in 1962 by the Japanese astronomer Kozai when he analyzed the orbits of asteroids [9]. For the orbit of a celestial body with



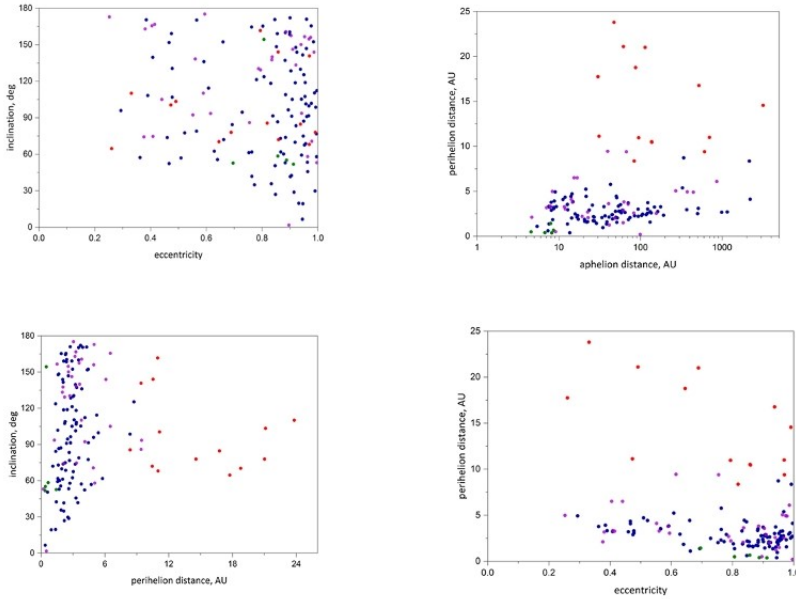
**Fig. 1.** The cumulative distribution of orbital inclinations for 144 Damocloids (black curve), 155 Centaurs (green curve), compared with 667 Jupiter family comets (brown curve) ( $P \leq 20$  years and inclinations  $20^\circ - 30^\circ$  or less), 401 LPCs (blue curve), and 76 Halley-type comets (magenta curve) ( $20 < P < 200$  yrs, and inclination from  $0^\circ$  to  $90^\circ$  or more), according to the JPL Horizons service, on 21st of February, 2018. The similarity between Damocloids, LPCs, and HFCs is seen, with even greater similarity between Centaurs and JFCs populations.

eccentricity  $e$  and inclination  $i$ , which revolves around a larger body, the following constant ratio is preserved:

$$\sqrt{(1 - e^2)} \times \cos i = \text{const}$$

That is, periodic oscillation can lead to resonance between two celestial bodies. Thus, nearly circular, extremely inclined orbits can gain a very large eccentricity in exchange for a smaller inclination. Thus, for example, increasing eccentricity, with a constant semimajor axis, decreases the distance between objects at perihelion, and this mechanism can force comets to become nearsolar.

As a rule, for objects in orbits with a small inclination, such fluctuations lead to precession of the pericenter argument. Starting from a certain minimum value of the angle - the Kozai angle, which is  $39.2^\circ$  for bodies in direct orbits and  $140.8^\circ$  for retrograde bodies - precession goes into libration around one of two values of the angle:  $90^\circ$  or  $270^\circ$ , that is, the pericenter (point of maximum convergence). will fluctuate around this value. Physically, the effect is related to the transfer of momentum and the preservation of its total amount in a connected system.



**Fig. 2.** The Damocloids' orbital parameters distributions with respect to the particles lifetime. Here red dots correspond to the particles from the long-lived subpopulation, the blue dots correspond to the particles from the short-lived subpopulation, and the violet dots are those particles for which less then a half of clones appeared in the long-lived subpopulation. The green dots are particles that broke the general rules, revealed by the plots analysis.

1. It was checked to what extent the distributions of inclined Damocloids, Centaurs and comets of individual families really differ [10]. For comparison, families with similar distributions were selected: Centaurs and JFC, as well as Damocloids, HTC and LPC. The Kolmogorov-Smirnov criterion was used [11].

2. Comparison of cumulative inclination-distributed and centaur comets of the known family (JFC). The distributions are based on 155 known centaurs and 667 JFCs for 18 inclination intervals with a step of  $10^\circ$  in the range  $0 - 180^\circ$ . The null hypothesis that their distributions are the same was tested. The maximum difference in values distributed for Centaurs - JFC is 0.146. Application of the Kolmogorov-Smirnov criterion shows that the probability of the same distribution is  $\alpha = 0.0094$ . That is, with a probability of more than 99%, these families differ.

3. Comparison of cumulative slope-distributed  $i$  Damocloids and known comets (NTC) and (LPC). The distributions are based on 144 known Damocloids, 76 NTCs, and 401 LPCs for 18 slope intervals with a step of  $10^\circ$  in the range  $0 - 180^\circ$ . The null hypothesis that their distributions are the same was tested.

The maximum difference between Damocloids and HTC is 0.212 . This gives the probability of the same distribution according to the Kolmogorov-Smirnov criterion  $\alpha = 0.023$ . That is, more than 97% of these Damocloids of the HTC family are different. The maximum difference in values distributed for Damocloids is LPC 0.134 . The application of the Kolmogorov-Smirnov criterion shows that the probability of the same distribution  $\alpha = 0.044$ . That is, the null hypothesis is rejected at the level of statistical significance above 95%. The maximum difference for LPC - HTC is 0.094. This gives the probability of the same distribution according to the Kolmogorov-Smirnov criterion  $\alpha = 0.65$ . In this case, there is an ambiguity when it is impossible to say definitely that LPCs and HTCs have the same or different distributions on the slope. Perhaps, with the further accumulation of data on new comets, this question will be clarified.

Thus, there are statistically reliable differences in the slope distributions between Damocloids, Centaurs, and comets with similar orbits on the other hand. Apparently, this indicates a different origin of these families.

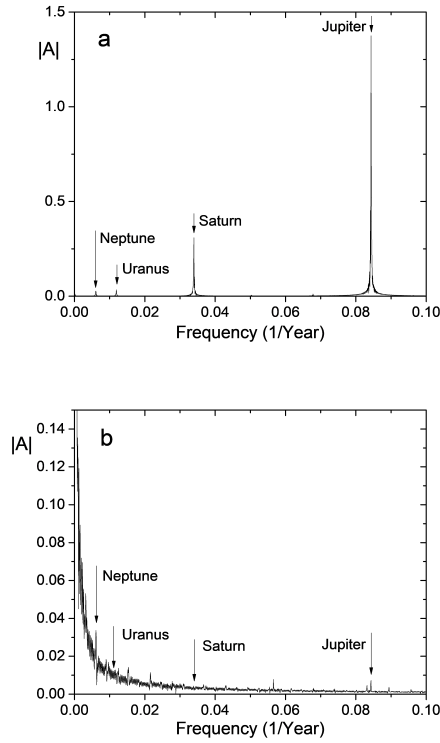
### 3. FOURIER-ANALYSIS

For some Damocloids, the dependence of the inclination  $i$  on time shows quasi-periodic changes. This is characteristic of the presence of resonances with giant planets. The presence of such resonances should manifest itself in the frequency spectrum of changes in the elements of the orbits over time. To detect such frequencies, we used the discrete Fourier transform method, which is widely used for digital signal processing. In this method, the signal (in our case, one of the elements of the orbit) is represented as a series  $x_n = \frac{1}{N} \cdot \sum_{k=0}^{N-1} A_k \cdot \exp\left(\frac{2\pi i}{N} kn\right)$ , where  $x_n$  is the value of the element of the orbit at the  $n$ th time,  $N$  is the total number of values of the element of the orbit,  $A_k$  is the complex amplitude for the  $k$ -th harmonic. Period  $P_k$  and frequency  $v_k$  for the  $k$ -th harmonic

$$P_k = \frac{k}{N} \cdot \Delta t, \quad v_k = \frac{1}{P_k} = \frac{N}{k \cdot \Delta t},$$

where  $\Delta t$  is the time step in Julian years for the calculated values of the orbit elements. The fast Fourier transform program from the Matlab mathematical package was used to study the frequency spectrum in the dependence of the elements of the orbits on time. Based on the values of the complex amplitudes, a power spectrum was constructed, which is the dependence of the modulus of the complex amplitude  $A_k$  on the frequency  $v_k$ . At the same time, the value of  $A_0$  was not taken into account, since we were interested in the change of orbital elements. As a rule, the spectrum shows frequencies that correspond to the rotation periods

of the giant planets  $0.08431 \text{ h}^{-1}$  (Jupiter),  $0.03393 \text{ h}^{-1}$  (Saturn) and less often with smaller amplitudes  $0.01190 \text{ h}^{-1}$  (Uranus),  $0.00607 \text{ h}^{-1}$  (Neptune).

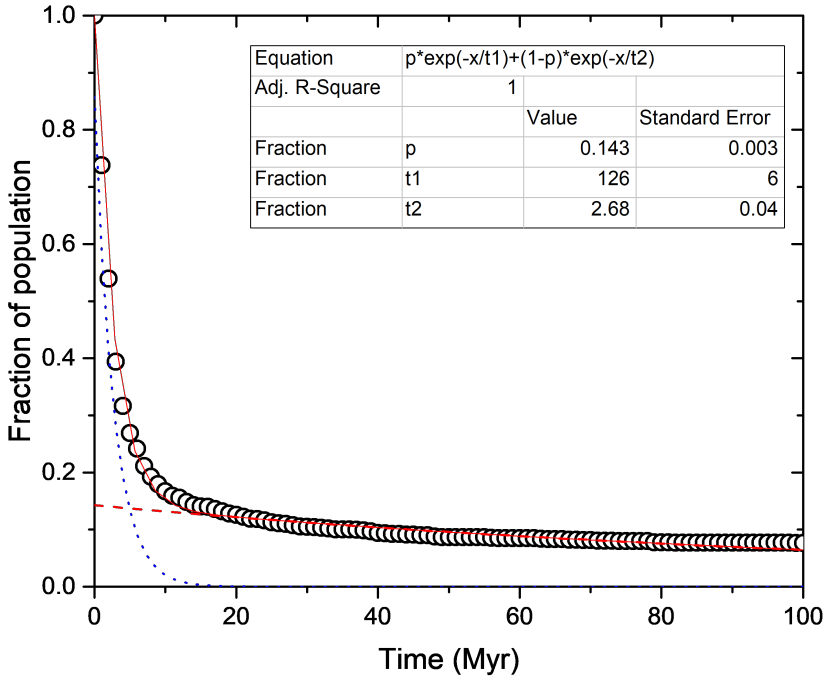


**Fig. 3.** Examples of spectra of changes in the inclination of the orbit  $i$  for Damocloids: a) with strong resonances with Jupiter and Saturn of Damocloid #418; b) in the absence of resonances with the giant planets of Damocloid #421.

#### 4. METHODS AND RESULTS

To perform the objectives of this work we accomplished the following steps:  
 1. We integrated the 1872 test particles (of 144 Damocloids and 12 clones for each of them, created by rounding up back and forth the last decimal number in the values of their state vector parameters) for 100 Myr forward in time, with the integration step 29.22 days and step of data output 80 years. The four giant planets and the Sun (with added mass of terrestrial planets) were taken into account. Each particle was followed till it reached 5000 AU distance from the Sun. RMVS integration technique [7] from SWIFTER software package [6] was used. Integration allowed us to get the particles' distribution on their lifetime

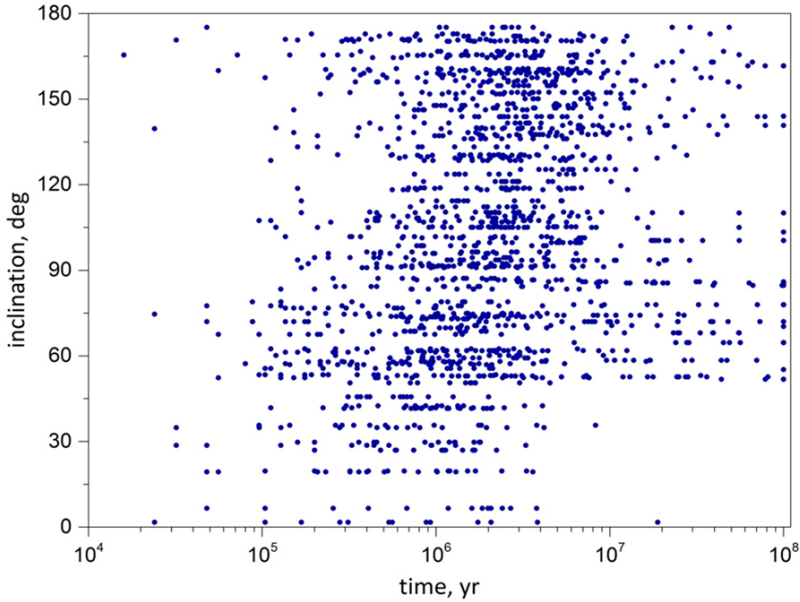
(Fig.4), and also distribution of particles' lifetimes as a function of their initial inclinations (Fig. 5). Also, in the course of calculations we analyzed all possible close encounters of Damocloids with planets in order to better understand the evolution of this family [12,13]. The decay half-life for the population is given by



**Fig. 4.** The fraction of surviving in the simulation Damocloids as a function of simulation length, up to 100 Myr forward in time (circles). The exponential law approximation was applied to the curve. Dotted line is the best fit for the short-lived subpopulation, while dashed line is the best fit for the long-lived subpopulation. Parameters of exponential equation and value errors are presented in a table within the plot.

$N = N_0 e^{-\lambda t}$ , where  $N_0$  is initial number of particles, and  $\lambda = \frac{0.611}{T_{1/2}}$ , where  $T_{1/2}$  is the half life time.

2. To reveal which orbital parameter is responsible for the length of particle's lifetime and the division of Damocloids in two subpopulations - long-lived and short-lived - we plotted the initial orbital parameters' distributions for 144 Damocloids, partially shown on Fig. 2. The orbital parameters distributions on the plots  $i(e)$ ,  $i(a)$ , and  $i(Q)$  don't reveal the division of Damocloids in two subpopulations by their lifetime. The orbital parameters distributions on the plots  $e(a)$ ,  $q(a)$ ,  $q(Q)$ ,  $i(q)$ ,  $q(e)$  do reveal the division of Damocloids in two subpopulations by their lifetime. Thus, by location of the Damocloids' particle in



**Fig. 5.** The particles' lifetimes distribution as a function of initial inclinations. Particles and clones on less inclined orbits live shorter, while particles on more inclined orbits live longer.

parameter spaces  $e(a), q(a), q(Q), i(q), q(e)$  it is possible to predict whether the particle belongs to the short-term or the long-term subpopulation. The long-lived subpopulation particles may have an arbitrary eccentricity, but they have  $q$  in the vicinity of Saturn's orbit or beyond, aphelia above 30AU (in the vicinity of Neptune's orbit and beyond), and semi-major axis  $a$  above 20 AU. Their inclinations are above 60 degrees, but not all particles with such inclinations are long-lived. The stricter division of subpopulations is by perihelion distance  $q$ . If  $q$  is about 9AU or above - we may predict without simulations that the particle most likely will be long-lived.

Five strange particles (green dots on Fig. 2); that broke the rules of division in two subpopulations, have something in common. These particles (343158 (2009 HC82), 2005 MW9, 2012 US136, 2010 CA55, and 2016 EB157) all have inclinations above 50 degrees, high eccentricities (0.696-0.912), and small perihelia, mostly inside Earth's orbit (0.377 AU, 0.402 AU, 0.489AU, 0.668AU, and 1.428 AU). But the sample is not big enough to make any profound conclusion.

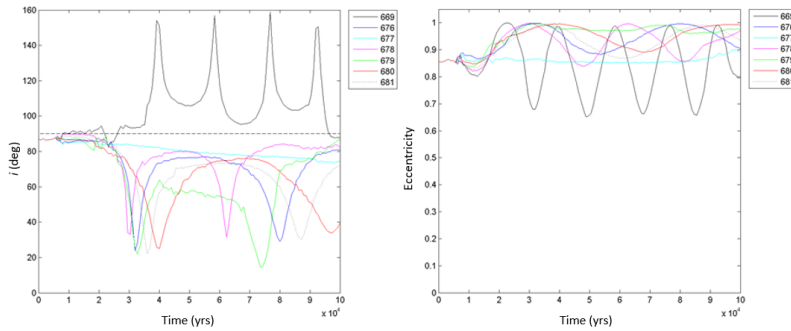
3. As a next step we applied K-S statistical test to these groups of particles (marked in red, blue, violet, and green on Fig. 2) and revealed that by  $i$  and  $e$  they are identical, but the statistically significant difference appeared for  $q$  (the highest),  $Q$ , and  $a$ .



4. The K-S test applied to the curves on Fig. 1 revealed that the closest to each other are populations of HTC's and LPC's, then Centaurs and JFC's, and then Damocloids and HTC's.

5. The dynamical evolution integration of Damocloids performed as a step 1 allowed to plot the orbital changes and to see their character. Some Damocloids revealed quasiperiodic behavior, others experienced transitions from direct to retrograde motion or vice versa, others got stuck in resonances.

6. The transiting particles follow the Lidov-Kozai mechanism. Below are the respective plots for  $i(t)$  (left) and  $e(t)$  (right) for same sets of particles (Fig. 6).



**Fig. 6.** The comparison of respective plots for  $i(t)$  (left) and  $e(t)$  (right) for same sets of particles. This figure is an attempt to show the Lidov-Kozai mechanism on the example of 7 individual particles.

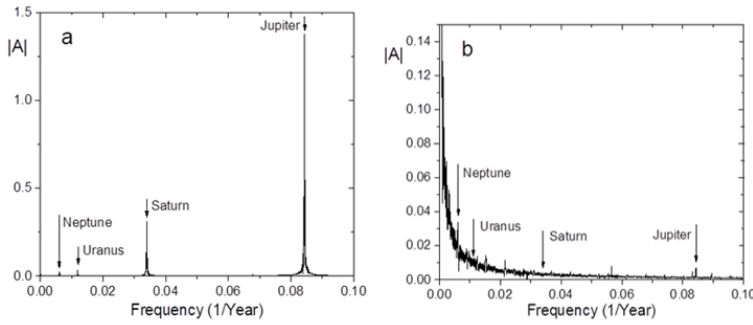
7. The long-lived survivals are captured in resonances with giant planets.

8. The Fourier analysis of the power spectrum of  $i$  shows which giant planet is responsible for Damocloids' resonances or dramatic changes in their orbits. Below are examples for strong resonances with Jupiter and Saturn (left, Fig. 7a, particle #418), and without resonances (right, Fig. 7b, particle #42).

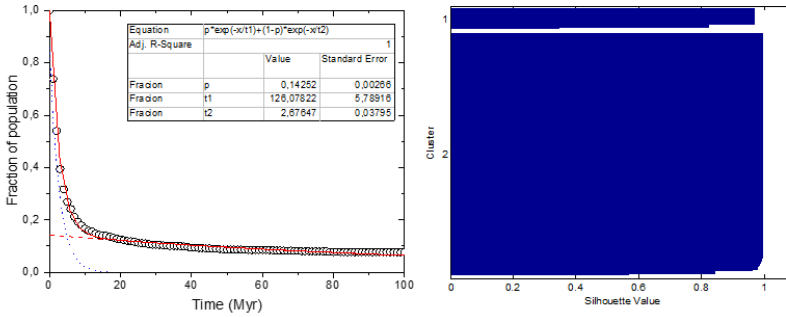
## 5. RESULTS

1. The Damocloid population maintains large orbital inclinations over an integration time of 100 million years.

2. Damocloids can be divided into two subpopulations, with a short and a long life, according to the dynamic life span. The first includes 85.75% of objects and has an average lifetime of  $2.68 \pm 0.04$  million years. The long-lived subpopulation accounts for 14.25% of objects, with an average dynamic lifetime of  $126 \pm 6$  million years (Fig. 8).



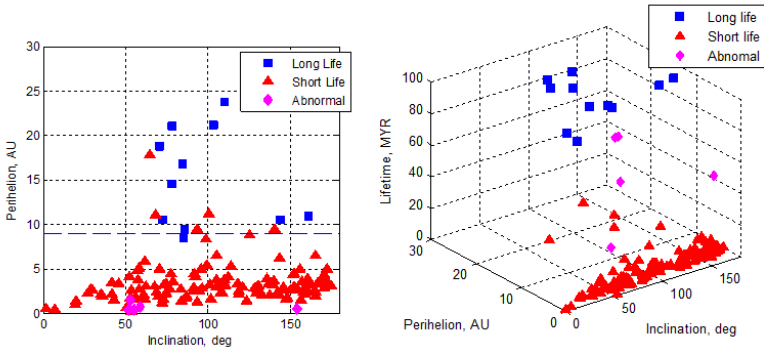
**Fig. 7.** The power spectrum of  $i$  in case of strong resonances with Jupiter and Saturn (left, Fig. 7a, particle #418), and without resonances (right, Fig. 7b, particle #421).



**Fig. 8.** Bimodal distribution of Damocloids according to dynamic during life, based on the results of approximation by exponential curves (left), and according to the results of cluster analysis (right)

3. It was analyzed which orbital parameter is responsible for the separation of Damocloids into two subpopulations. The separation between them is clearly visible on the graphs  $e(a)$ ,  $q(a)$ ,  $q(Q)$ ,  $i(q)$ ,  $q(e)$ . The main parameter by which it is possible to determine whether Damocloid belongs to a subpopulation with a longer or shorter life time without prior integration is the perihelion distance, less or more than 9 AU. (Fig. 9).

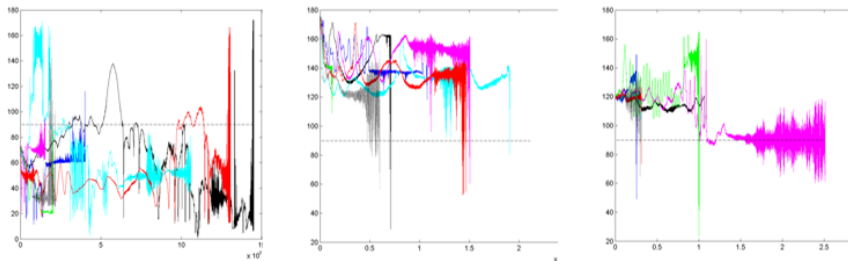
4. Found 5 objects that violate the rule of point 3, because they have a long-life time, and on the graphs they occupy a position among objects with a short life time. It is shown that the transition of the orbital inclination through a value of 90 degrees, i.e., from forward motion to reverse motion or vice versa, sometimes occurs according to the Lidoy-Kozai mechanism, when the inclination is "exchanged" for the eccentricity, i.e., a change in the inclination leads to a



**Fig. 9.** Two-dimensional (left,  $q(i)$ ) and three-dimensional (right, + particle lifetime) graphs showing the distribution of Damocloids into two subpopulations by lifetime. Blue squares – particles with a long lifetime, red triangles – with a short lifetime, purple rhombuses – anomalous particles (with a long lifetime, but in parametric space they occupy positions among particles with a short lifetime).

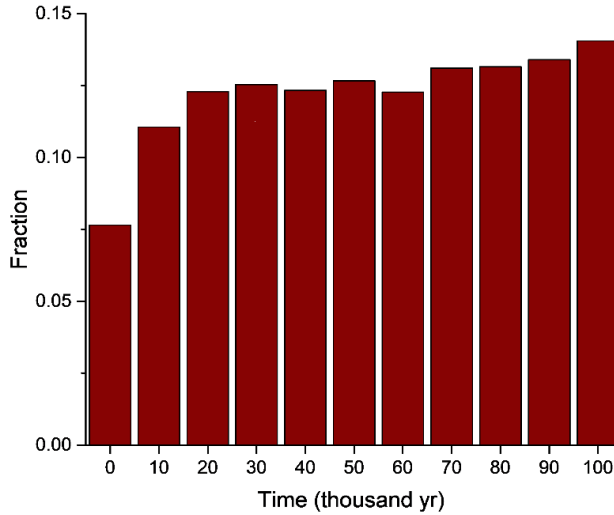
sharp change in the value of the eccentricity of the orbit.

5. During the integration time of 20 million years forward, out of 144 sets of orbits (for each known body and its 12 clones), only 17 sets did not have a single body or clone crossing the  $90^\circ$  orbital inclination mark. Some objects undergo repeated inclination changes and transitions through 90 degrees (Fig. 10, left), while others leave the integration zone (from the Sun to 5000 AU ) after the inclination change (Fig. 10, middle). Among the Damocloid population are objects that undergo libration around an angle of  $90^\circ$  (Fig. 10, right).



**Fig. 10.** An example of the orbital evolution of bodies that: repeatedly switched from direct motion to retrograde or vice versa (left); after changing the inclination, they leave the integration zone (from the Sun to 5000 AU) (middle); undergo libration around an angle of  $90^\circ$  (right).

6. Over time, Damocloids can replenish the space near the Earth's orbit. A preliminary analysis gave an estimate of the rate of transition of objects from the Damocloid population to NEO orbits – 0.11% of the population in 10 million years. Integration with a smaller step of 100,000 years showed that every 10,000 years, at least temporarily, from 7.64% to 14.05% of the population has a low perihelion in the region less than or equal to 1.3 AU. 7. Taking into account the bimodal



**Fig. 11.** The portion of the Damocloids that during the integration time of 100,000 years had a perihelion in the region of  $q \leq 1.3$  AU. (data for every 10 thousand years).

distribution of Damocloids according to their lifetime, it is proposed to refine the definitions of Damocloids according to their perihelion distance  $q$  (more or less than 9 AU). It has been proposed to call these subpopulations with longer and shorter life spans trans-Saturnian and pre-Saturnian Damocloids, respectively.

## 6. CONCLUSIONS

1. The simulation revealed the existence of two subpopulations of Damocloids: 85.75% are short-lived with the median dynamic lifetime of  $2.68\text{Myr} \pm 0.04\text{Myr}$ , and 14.25% of the population is long-lived with dynamic lifetime  $126 \pm 6\text{Myr}$ . The main parameter by which the division can be made without prior simulations is perihelion distance: Damocloids with  $q$  from 9AU and above will most likely belong to the long-lived subpopulation. The longlived survivals are captured in resonances with giant planets.

2. Taking into consideration the bimodality of Damocloids lifetime distribution we propose to add to the definition of Damocloids by two criteria (asteroids, and  $T_J \leq 2$ ) the indication to which subpopulation the object belongs - long-lived or short-lived - by perihelion distance  $q$  (more or less than 9AU ). We propose to name these long-lived and short-lived subpopulations as trans-Saturnian and pre-Saturnian Damocloids.

3. Five strange particles were revealed. They are long-lived, but occupy the parameter space region typical for the short-lived particles. These strange particles all have inclinations above 50 degrees, high eccentricities (0.696-0.912), and small perihelia, mostly inside Earth's orbit (0.377 AU, 0.402 AU, 0.489 AU, 0.668 AU, and 1.428 AU).

4. Some of the evolutionary tracks allow transitions from retrograde motion to direct and vice versa. The Damocloids population retains members on retrograde orbits over the course of integration.

5. Some of Damocloids reached smaller perihelia over the course of integration, allowing them to become nuclei of Halley-type comets.

6. Statistically the closest to each other are populations of HTC's and LPC's, then Centaurs and JFC's, and then Damocloids and HTC's.

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