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IMPACT OF THE COSMOLOGICAL CONSTANT ON THE EVENT HORIZON

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Schwarzschild's solution for Einstein's equations in the case of a non-zero cosmological constant is considered. Observations show that the cosmological constant is different from zero. Therefore, the study of such solutions is necessary. It is shown that the dependence of the event horizon on the cosmological constant is discontinuous. The event horizon exists for zero and negative values of the cosmological constant of the central spherically symmetric body.

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1. INTRODUCTION

According to Einstein's concept, space-time geometry is determined not a priori, but by the motion and distribution of matter, and gravitational phenomena were identified with the non-Euclidean geometric structure of space-time. Einstein showed that the local principle of equivalence of gravitational and inert mass in combination with the principle of relativistic invariance with the necessity of leads to field theory and Riemann's geometry$^{)}$, and the metric tensor components play the role of gravitational potential, [1, 8, 9]:

$$R_{ik} - \frac{1}{2} g_{ik} R = \varkappa T_{ik}. \tag{1}$$

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$^{)}$More precisely, pseudo-Riemannian geometry, since the $g_{ik}-$metric of space-time has the signature $(+1, -1, -1. -1)$. Only geometries with a positive-definite quadratic form are called Riemann manifolds.

The constant $\varkappa$ is related to the gravitational constant k as the following relationship:

$$\varkappa c^2 = 8\pi k/c^2 = 1,87 \cdot 10^{-26}\mathrm{m/kg},$$

where $c-$ is the speed of light in a vacuum.

After the creation of the foundations of the general theory of relativity, A. Einstein proceeded to solve cosmological problems. In connection with cosmological problems, he proposed gravitational equations with the cosmological "$\Lambda-$term", [1,4,9]:

$$R_{ik} - \frac{1}{2}\, g_{ik}R + \Lambda g_{ik} = \varkappa T_{ik}. \tag{2}$$

Observations show that the cosmological constant Λ is different from zero. Therefore, the study of such solutions is necessary.

2. SCHWARZSCHILD'S SOLUTION AND THE GRAVITATIONAL RADIUS

The first exact solution was found by Schwarzschild in 1916. The linear element according to the Schwarzschild solution has the form, [3]:

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{dr^2}{1 - 2M/r} - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{3}$$

As can be seen, this solution represents a static and spherically symmetric field.

A similar solution for a static and spherically symmetric field can be written in the case of the Einstein equation eq.(2) with a cosmological term, [3]:

$$ds^2 = \left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right) dt^2 - \frac{dr^2}{1 - 2M/r - \Lambda r^2/3} - r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right). \tag{4}$$

These solutions are called the outer Schwarzschild solutions.

The Schwarzschild solution with $\Lambda = 0$, eq.(3) has a singularity at radius and valid only outside this radius:

$$r_g = 2M. \tag{5}$$

Since the parameter r_g has the dimension of mass, both solutions are interpreted as a field surrounding the gravitating body, and r_g is called its gravitational or Schwarzschild radius.

As long as the real radius of the body is greater than its gravitational radius, there are no obstacles to applying the Schwarzschild solution$^{)}$. But at present,

$^{)}$For the Sun, the Schwarzschild radius $r_{0\odot} = 2.95\ km$, i.e. is deep within it. For the Earth, the gravitational radius $r_{g\oplus} = 8.87\ mm$. And for the proton, the Schwarzschild radius cosmological objects are known to be entirely within their Schwarzschild radius. They are called *Black holes*. It is known for certain that at the centers of galaxies and quasars, as a rule, there are black holes, even binary systems of black holes.

The Schwarzschild radius is not a singularity and is called the *event horizon* or the *surface of a black hole* and is also called the *static limit*. The fate of the collapsing matter after it crosses the surface of the black hole is unknown. No radiation or matter can escape a black hole.

The Schwarzschild solution corresponds to a non-rotating non-charged black hole.

Similarly, the equation for the gravitational radius at a non-zero cosmological constant is determined by the singularity of the solution from (4):

$$1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2 = 0. \tag{6}$$

This equation is reduced to an equation of the third degree:

$$\Lambda r^3 - 3r + 6M = 0$$

and is solved by the Cardano method, [2], (see Appendix).

For $\Lambda \neq 0$ it has two complex and one real solution. We are, of course, interested in a real solution that has a physical meaning. By this solution, the dependence of the gravitational radius on the cosmological constant is given by a discontinuous function of the following form:

$$r_g = \begin{cases} 2M, & \text{if} \quad \Lambda = 0,\ M \neq 0, \\ \kappa/\Lambda + 1/\kappa, & \text{if} \quad \Lambda \neq 0, \end{cases} \tag{7}$$

where

$$\kappa = \left(-3M\Lambda^2 + \Lambda\sqrt{9M^2\Lambda^2 - \Lambda}\right)^{1/3}.$$

By direct substitution, one can verify that (7) is a solution to (6).

The condition of positivity of the gravitational radius r_g is possible in two cases:

1. if $\Lambda < 0$ then $M > 0$;
2. if $\Lambda > 0$ then $9M^2\Lambda > 1$ and $M < -M_g < 0$, where

$$M_g \equiv \frac{1}{3\sqrt{\Lambda}}. \tag{8}$$

We remind you that M is the constant of integration of the Einstein equations.

is 10^{-59} *cm*, which is 37 orders of magnitude smaller than its characteristic radius of 10^{-13} *cm*, [9].

3. CONCLUSIONS

Further analysis of the obtained dependence of the gravitational radius on the cosmological constant leads to interesting conclusions. Obvious physical considerations require that the gravitational radius be a positive value: $r_g > 0$.

The parameter M takes positive values for the Schwarzschild interval without the cosmological term (3) and is interpreted as the mass of the gravitating body.

For a Schwarzschild interval with a cosmological term (4), negative values of M lead to a different interpretation. As shown in [5–7], negative mass or energy, which is the same, can be interpreted as *Dark energy*, which is a *Cosmological quantum effect*. Therefore, this solution cannot be excluded when considering quantum effects in cosmology.

APPENDIX

In the appendix, we present a direct application of the Cardano method to eq.(6).

In case $\Lambda \neq 0$, eq.(6) can be reduced to the form:

$$r^3 - \frac{3}{\Lambda} r + \frac{6M}{\Lambda} = 0. \tag{9}$$

To further simplify this equation, we introduce a couple of new variables (u, v) using the substitution $r = u + v$:

$$u^3 + v^3 + \left(3uv - \frac{3}{\Lambda}\right)(u + v) + \frac{6M}{\Lambda} = 0$$

An additional condition can be imposed on variables (u, v):

$$3uv - \frac{3}{\Lambda} = 0.$$

Then for the unknowns (u, v) we obtain the following equations:

$$u^3 + v^3 = -\frac{6M}{\Lambda}; \qquad u^3 v^3 = \frac{1}{\Lambda^3}. \tag{10}$$

According to the theorem of Viet, variables (u^3, v^3) are solutions of the following square equation:

$$x^2 + \frac{6M}{\Lambda} x + \frac{1}{\Lambda^3} = 0. \tag{11}$$

The solutions of the quadratic equation (11) are well known:

$$u^3 = -\frac{3M}{\Lambda} + \sqrt{\left(\frac{3M}{\Lambda}\right)^2 - \frac{1}{\Lambda^3}},$$
$$v^3 = -\frac{3M}{\Lambda} - \sqrt{\left(\frac{3M}{\Lambda}\right)^2 - \frac{1}{\Lambda^3}}. \tag{12}$$

From the last levels one can obtain an explicit expression for the gravitational radius $r = u+v$. When taking cube roots, it is necessary to ensure that conditions (10) are met. We omit the details of the calculations.

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