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QUANTUM-MECHANICAL DESCRIPTION OF MATTER IN COMPACT STARS

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ABSTRACT
An analytical study presents the physical phenomena occurring in superdense stars, with particular attention to the structure and properties of the crystalline crust. Numerical estimates of the density and lattice parameters of matter in the crust are provided. It is shown that under extreme conditions of density, as realized in compact stellar objects, the matter forms a degenerate electron gas whose properties are governed by quantum-mechanical principles.

Key words: compact objects – neutron stars – dense matter – degenerate matter

1 INTRODUCTION

Compact stars are astronomical objects formed as end products of stellar evolution, characterized by extremely high densities and small radii. They are the remnants of massive stars that have exhausted their hydrogen fuel and undergone subsequent nuclear burning stages, including helium fusion, leading to the explosive event of a supernova. The explosion leaves behind a dense central remnant surrounded by an extended nebula. The class of compact objects includes neutron stars, white dwarfs, and black holes (Shapiro & Teukolsky 1983; Yakovlev 2016).

The principal distinction between compact and normal stars lies in their density and size. Compact stars have dimensions much smaller than those of main-sequence stars of comparable mass. Neutron stars are of scientific interest because they serve as natural laboratories for studying matter under extreme physical conditions. They allow testing of theoretical models of superdense matter by comparing observational data with the predictions of particle interaction theories (Yakovlev 2016).

Although neutron stars have been studied for decades, their internal structure remains incompletely understood. Some models describe a relatively simple composition with a thin outer crust over a layer of neutron fluid, while others propose a more complex stratification. In such models, the outer crust comprises several layers differing in density and nuclear composition, with a crystalline layer of iron on the surface, beneath which lies degenerate matter. The core may consist of exotic phases such as quark–gluon plasma or pion condensate (Haensel et al. 2007; Yakovlev 2016).

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2 OBSERVATION AND RESULTS

At extreme densities ranging from 10^4–10^7 g cm^{-3} in white dwarfs to above 10^9 g cm^{-3} in neutron star crusts, matter ceases to obey classical thermodynamic behavior. Instead, its properties are dominated by the Pauli exclusion principle, a cornerstone of quantum mechanics (Lattimer & Prakash 2004; Takibayev & Boshkayev 2017).

This principle forbids identical fermions (electrons) from occupying the same quantum state, resulting in the formation of a degenerate electron gas—a state in which pressure arises solely from the filling of energy levels up to the Fermi energy. Such degeneracy pressure is the fundamental component of the equation of state of compact stars.

Degeneracy occurs when the average interparticle distance becomes comparable to the electron de Broglie wavelength, necessitating a full quantum-statistical treatment via the Fermi–Dirac distribution (Landau & Lifshitz 1976; Takibayev & Boshkayev 2017).

The Fermi–Dirac distribution for fermions at temperature T is given by:

$$f(E) = \frac{1}{e^{\frac{E-\mu}{kT}} + 1} \qquad (1)$$

where μ is the chemical potential.

In the limit $T \to 0$, $f(E) = 1$ for $E < E_F$, and $f(E) = 0$ for $E > E_F$, meaning all states below the Fermi level are occupied.

Considering a system of noninteracting electrons in volume V, each quantum state is characterized by momentum p. The number of states per phase-space element is limited to

$$\frac{2}{h^3}\, d^3p\, d^3r,$$

where the coefficient 2 reflects the two spin states of the electron.

Let the number of electrons within a volume V be N, and the electron number density be:

$$n_e = \frac{N}{V}. \tag{2}$$

At $T = 0$, the electrons occupy all quantum states from $p = 0$ up to the maximum momentum, the Fermi momentum p_F, which is determined by the condition:

$$N = \frac{2V}{h^3} \int_0^{p_F} 4\pi p^2 \, dp. \tag{3}$$

Performing the integration, we obtain:

$$n_e = \frac{8\pi}{3h^3} {p^3}_F. \tag{4}$$

Hence, the Fermi momentum is given by:

$$p_F = h(3\pi^2 n_e)^{1/3}. \tag{5}$$

This fundamental quantity determines all the physical properties of the degenerate gas. The energy of an electron with momentum p in the non-relativistic regime takes the form:

$$E = \frac{p^2}{2m_e}, \tag{6}$$

while in the relativistic regime it is given by

$$E = \sqrt{p^2c^2 + m_e^2c^4} - m_e c^2. \tag{7}$$

Fermi energy:

$$E_F = E(p_F). \tag{8}$$

For typical white-dwarf densities, the Fermi energy is $E_F \sim 0.1$–2 MeV, which often exceeds the thermal energy kT, thereby leading to degeneracy.

Degeneracy occurs when

$$kT \ll E_F. \tag{9}$$

For white dwarfs, the typical temperature is $T \sim 10^7$ K, while the Fermi temperature is $T_F \sim 10^{10}$ K; hence, $T/T_F \sim 10^{-3}$–10^{-4}.

Temperature corrections are therefore negligible, and the pressure is determined almost entirely by quantum effects rather than by thermal motion.

The theoretical foundation of neutron-star models is based on the quantum theory of matter and on the concept of a degenerate Fermi gas of spin-$\frac{1}{2}$ particles. However, many aspects concerning the internal structure of neutron stars, as well as the nuclear reactions and microscopic processes occurring within them, remain insufficiently understood. Therefore, the further development of neutron-star physics is closely associated with the formulation of new theoretical approaches and models, as well as the discovery of novel astrophysical phenomena.

Let us consider the state of matter in the outer layers of neutron stars and note the main characteristics of these regions. They represent superdense crystalline structures whose density increases rapidly with depth, i.e., with increasing gravitational pressure. The outer crust constitutes an extremely dense crystal that, with depth, transforms into a Coulomb lattice immersed in a sea of degenerate electrons. Under the action of intense pressure, atoms lose their bound electron shells and remain as "bare" nuclei fixed at the lattice sites, embedded in the degenerate Fermi liquid of electrons (Takibayev & Boshkayev 2017; Takibayev et al. 2013). Beginning at densities $\rho > 1.2 \times 10^7$ g cm^{-3}, inverse beta-decay reactions become energetically possible:

$$e^- + p \to n + \nu_e.$$

The inner crust is defined as the region where the process of neutronization of atomic nuclei begins, that is, nuclei become increasingly neutron-rich. The matter in this region consists of an ultrarelativistic Fermi gas of degenerate electrons, a lattice of "bare" nuclei, and a population of free neutrons.

However, in this region the neutrons still form a non-relativistic Fermi gas. At greater depths, beginning from densities of the crystalline lattice exceeding $\rho > 3 \times 10^{13}$ g cm^{-3}, the contribution of the neutron Fermi gas to the total pressure starts to dominate over that of the electron component.

Let us introduce several quantities and definitions essential for the subsequent discussion. The de Broglie wavelength of a neutron is given by

$$\lambda = \frac{h}{\sqrt{2m_n E_n}}, \tag{10}$$

or equivalently,

$$\lambda_n = \frac{2\pi\hbar}{\sqrt{2m_n E_n}}. \tag{11}$$

If $E_n \approx 0.1$ MeV, then $\lambda \approx 90$ fm; for $E_n \approx 0.4$ MeV, $\lambda \approx 45$ fm; and for $E_n \approx 0.8$ MeV, we obtain $\lambda \approx 31.8$ fm.

The number density of neutrons can be related to the Fermi momentum if the neutrons are densely packed and form a degenerate Fermi gas:

$$n_n = \frac{2}{h^3} \int_0^{p_F} 4\pi p^2 \, dp = \frac{8\pi}{3h^3} p_F^3, \tag{12}$$

since $p_F = \hbar k_F$. Hence, for $E_F = E_n \approx 0.1$ MeV, one obtains $k_F \approx 0.0695$ fm^{-1}, which corresponds to a number density of the degenerate neutron Fermi gas of

$$n_n = \frac{8\pi}{3}(6.95 \times 10^{11})^3 \, \text{cm}^{-3} = 2.8 \times 10^{36} \, \text{cm}^{-3}, \tag{13}$$

and an average inter-neutron spacing of $r_n \approx 7.2$ fm.

Let us now return to the region of interest within the neutron star, namely the outer and inner crusts, where the densities lie in the range 1.2×10^7 g cm$^{-3} < \rho < 1.4 \times 10^{14}$ g cm^{-3}. In this region, the most characteristic feature is the presence of crystalline structures, which represent the energetically most favorable configuration of matter in this density range (Takibayev & Boshkayev 2017).

3 CONCLUSIONS

In the crustal layers of neutron stars, the dominant forces are gravitational and Coulomb, which act in opposition to one another. Gravitational pressure tends to compress the matter, while the degeneracy pressure of the electron Fermi gas counteracts this compression, since the electrons in the lattice are densely packed.

The electron component of the matter is directly related to the mass density, because the number of electrons must exactly compensate the positive charge of the "bare" nuclei in the lattice.

Thus, in each layer of the crust—and even within every unit cell of the crystalline lattice—charge neutrality must be maintained in equilibrium. This condition allows the thermodynamic characteristics of the matter to be expressed as functions of the density.

The mass density of matter can be related to the interionic spacing in the lattice by the relation

$$\rho = n m_i, \qquad (14)$$

where m_i is the mass of nuclei of species i.

From Eq. (10), one can express the nuclear concentration in the crust of a neutron star for different density values as

$$n_A = \frac{\rho}{m_A}, \qquad (15)$$

and the distance between neighboring nuclei occupying the lattice sites is determined by

$$d = \frac{1}{n_A}. \qquad (16)$$

Let us illustrate these parameters using examples of specific nuclei—phosphorus and sulfur. Assuming that the lattice is composed of phosphorus nuclei at a density of $\rho = 10^6$ g cm^{-3}, we obtain a concentration of $n = 0.19 \times 10^{29}$ cm^{-3}. In this case, the corresponding interionic distance between phosphorus nuclei is $d = 3.7 \times 10^3$ fm.

For sulfur nuclei occupying the lattice sites of the neutron-star crust, the concentration is found to be $n = 0.18 \times 10^{29}$ cm^{-3}, and the distance between neighboring nuclei is $d = 3.8 \times 10^3$ fm.

The same parameters were also calculated for densities of $\rho = 10^7$ g cm^{-3}, $\rho = 10^9$ g cm^{-3}, $\rho = 10^{11}$ g cm^{-3}, and $\rho = 10^{13}$ g cm^{-3}.

Based on the obtained data, a simple graph illustrating the dependence of density on the inter-nuclear distance was constructed. It should be noted that the nuclei considered here, $^{31}_{15}$P and $^{32}_{16}$S, differ only slightly in mass.

In Figure 1, the black line shows the dependence $\rho \propto d^{-3}$, demonstrating the smooth increase in density with decreasing inter-nuclear distance. In modeling the crystalline structure of matter in the neutron-star crust, density values corresponding to different depths were used. The analysis revealed that the matter density increases monotonically with depth, accompanied by a decrease in the lattice parameter. This behavior is consistent with the increasing degree of degeneracy and the progressive compression of the crystal lattice.

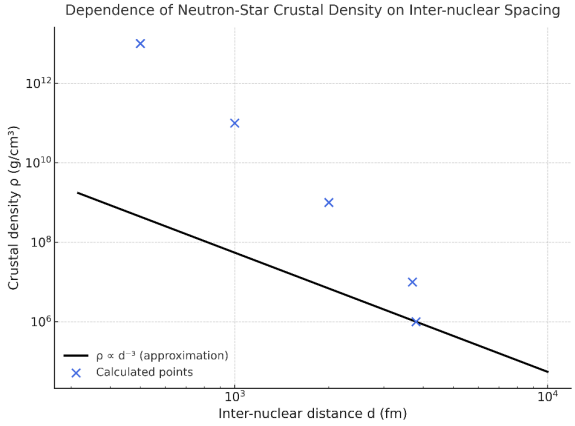

Figure 1. Dependence of the crustal density of a neutron star on the inter-nuclear spacing in the superdense lattice

An important consequence of variations in the parameters of the crystal lattice in the crust of a neutron star is their impact on the transport properties of matter, such as thermal conductivity, electrical conductivity, and viscosity. In contrast to the simple geometric relation $\sim d^{-3}$, these characteristics are governed by microscopic particle scattering processes and depend on the quantum properties of a degenerate Fermi gas.

Under the conditions prevailing in the crust of a neutron star, degenerate electrons are the primary carriers of heat and electric current. The transport coefficients of a degenerate electron gas in the neutron star crust can be described within the framework of kinetic theory (Landau & Lifshitz 1976), where the Fermi momentum p_F, the Fermi velocity v_F, and the collision frequency ν play a key role.

The thermal conductivity of a degenerate electron gas may be approximated as:

$$k \sim \frac{1}{3} C_v v_F^2 \tau, \qquad (17)$$

where C_v is the electron heat capacity, v_F is the Fermi velocity, and

$$\tau = \frac{1}{\nu}$$

is the characteristic time between collisions. For a degenerate gas, the heat capacity is proportional to the temperature and the density of states:

$$C_v \propto n_e \frac{kT}{\varepsilon_F}, \qquad (18)$$

As the matter density increases, the Fermi momentum grows according to:

$$p_F \sim (3\pi^2 n_e)^{1/3} \propto \rho^{1/3},$$

which leads to an increase in the Fermi velocity and, consequently, to an enhancement of thermal conductivity. However, this increase is not monotonic, since a decrease in the

Table 1. Estimate of transport properties of neutron star matter

Regime	ρ, g/cm^3	n_e, cm^{-3}	$\frac{p_F}{m_e c}$	σ, s^{-1}	k
Relativistic	10^9	3×10^{32}	~ 8	10^{22}	10^{20}
Ultrarelativistic	10^{13}	3×10^{36}	~ 80	10^{26}	10^{24}

lattice spacing strengthens electron scattering by lattice ions, as well as by impurities and structural defects.

The electrical conductivity of matter in the crust of a neutron star is given by:

$$\sigma \sim \frac{n_e e^2}{m_e^* \nu} \qquad (19)$$

where m_e^* is the effective electron mass. Similarly to thermal conductivity, an increase in density leads to a higher concentration of charge carriers; however, the concurrent rise in collision frequency at high densities limits the growth of conductivity.

Of particular interest is the transition from the non-relativistic to the relativistic regime of the electron gas.

At the high densities characteristic of the deeper layers of the crust, the condition

$$p_F \gg m_e c,$$

and the electron energy is given by

$$E \approx p_F c.$$

In this regime, the dependence of pressure on density, as well as the transport coefficients, is modified, which may lead to deviations from the simple power-law relations obtained in the non-relativistic approximation.

Let us present several numerical estimates for layers with densities $\rho = 10^9\, g/cm^3$ and $\rho = 10^{13}\, g/cm^3$, adopting the following initial assumptions: the electron fraction $Y_e \approx 0.5$ (order of magnitude), the nucleon mass $m_{\rm nucl} = 1.66 \cdot 10^{-24}\, g$, the electron number density $n_e = \frac{Y_e \rho}{m_{\rm nucl}}$, and $\nu = 10^{17}\, c^{-1}$, $T = 10^8 K$. The estimates of the transport coefficients demonstrate a significant increase in both electrical and thermal conductivity with increasing density. However, this increase is not strictly proportional to the growth in electron concentration, since compression of the crystal lattice leads to an enhanced scattering frequency.

Thus, the reduction of the inter-nuclear spacing within the crystal lattice with depth reflects not only the geometric compression of matter but also significantly affects the microphysical transport processes. This may lead to modifications in the thermal and electrical properties of the neutron star crust, which, in turn, can manifest in observable characteristics such as thermal evolution and X-ray emission.

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