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INVESTIGATION OF STRUCTURAL DESTRUCTION IN FILTRATION AND FLOW OF HETEROGENEOUS FLUIDS

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Several issues arising during the flow of heterogeneous fluids are related to changes occurring in their composition. Studying these changes within heterogeneous fluids is one of the crucial factors in assessing the structural stability at the interface of the constituent fluids. Therefore, various experiments related to the evaluation of the adsorption process in the filtration of dispersed systems have been conducted, and pressure-recovery curves have been constructed. Additionally, an algorithm has been developed for the numerical and asymptotic solutions of the equations describing the process, taking into account adsorption and convective diffusion during the filtration of solutions.

Keywords: *adsorption, pressure, concentration, diffusion, porous medium.*

INTRODUCTION

Mechanical and physicochemical changes occurring within many heterogeneous fluids significantly influence flow processes. In this context, the complex systems used can be considered as a unified thermobaric unstable medium impacting flow processes. Changes in solutions may manifest as rheological, sedimentation, thermodynamic, and other instabilities [1, 2, 9]

In order to conduct research in this direction, an experimental setup was created in the laboratory, and various properties of several water-based solutions were studied under thermobaric conditions. The conditions for their stable states were determined.

In oilfield mechanics, several unresolved problems are related to changes occurring within heterogeneous fluids during their flow. Studying such changes both within the composition itself and in contact with artificial additives can be crucial factors for regulating technological processes. It is possible to influence the structural instability of complex disperse fluids, which play an exceptional role in technological processes, and obtain the ability to modify this state [6]. Such conditions are encountered in drilling muds, cement slurries in well cementing, blood, gas-liquid mixtures, and so on.

FORMULATION OF THE PROBLEM

Solutions of polymers in water, containing suspended dissolved substances, undergo diffusion when chemical reagents are injected into oil reservoirs, interacting with the rock skeleton and reservoir fluids.

The most common types of material exchange occurring here are sorption and desorption processes. As a result of simultaneous sorption and desorption processes, macromolecules are distributed between the solution and the solid phase.

Polymer solutions adsorbed on the surface of the solid medium can alter the properties of the boundary layer structure, the compaction and aggregation of macromolecules, and other characteristics. Most importantly, the adsorption process plays a significant role in blocking high-permeability reservoir pores in oil fields [6, 7, 12, 13].

MATERIAL AND METHODS

Various experiments have been conducted to evaluate the adsorption process in the filtration of disperse systems in a porous medium. To evaluate this process, the pressure recovery curves in the medium model were observed and examined for the samples of the studied solution before and after filtration in the porous model under the same conditions, focusing on changes in concentration.

Experiments conducted to check the dependence of the adsorption process on changes in porosity also provide a basis for more accurately confirming these considerations. The pressure recovery curves obtained in the porous medium model can further validate this more accurately (Figure 1).

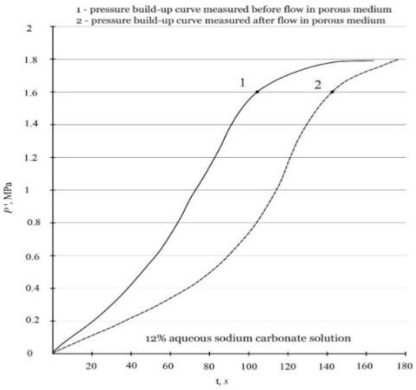

Fig. 1. Pressure build-up curve measure

The one-dimensional filtration equation, which combines the non-loss of polymer substances while considering convective diffusion and sorption kinetics, forms the basis of the mathematical model in this direction [8, 10, 11].

The adsorption process is considered to be in equilibrium, and its relation to concentration is given by the Henry isotherm. This equation, considering adsorption and convective diffusion with $\vartheta = \vartheta_0 = const$, is written in the following form [3-5].

$$\vartheta \frac{\partial U}{\partial x} + \frac{\partial (mU)}{\partial t} + \frac{\partial a_*}{\partial t} = D \frac{\partial^2 U}{\partial x^2} \qquad (1)$$

here

$\vartheta = const$ - $m = m(x,t)$- the filtration rate, which depends on the porosity of the medium;
$U = U(x,t)$ - the concentration of adsorption at a distance x from the beginning of the porous tube.
The following relationship exists between $m(x,t)$- the porosity and $U(x,t)$ - the concentration

$$m(x,t) = m_0 \pm \frac{\Gamma}{2\delta} U(x,t) \qquad (2)$$

here

Γ - Henry's coefficient;
δ - the density of the adsorbed model in the solid phase;
m_0 - initial porosity.

We can write the line according to the isotherm:

$$a_*(x,t) = \Gamma \cdot C_*(x,t) \qquad (3)$$

Suppose that,

$$C_*(x,t)=\frac{U(x,t)}{2} \tag{4}$$

We derive from equations (1) to (4):

$$\frac{\partial m}{\partial t}=\pm\frac{\Gamma}{2\delta}\frac{\partial U}{\partial t},$$

$$\frac{\partial(mU)}{\partial t}=\frac{\partial m}{\partial t}\cdot U+m\frac{\partial U}{\partial t}=\pm\frac{\Gamma}{2\delta}\cdot U\frac{\partial U}{\partial t}+m\cdot\frac{\partial U}{\partial t}=$$

$$=\left(\pm\frac{\Gamma}{2\delta}\cdot U+m\right)\frac{\partial U}{\partial t}=\left(\pm\frac{\Gamma}{2\delta}\cdot U+m_0\pm\frac{\Gamma}{2\delta}\cdot U\right)\frac{\partial U}{\partial t}=\left(m_0\pm\frac{\Gamma}{\delta}U\right)\frac{\partial U}{\partial t}$$

Then equation (1) is replaced by the following equations:

$$A(x,t)\frac{\partial U}{\partial t}=D\frac{\partial^2 U}{\partial x^2}-\vartheta\frac{\partial U}{\partial x} \tag{5}$$

here

$$A(x,t)=m_0+\frac{\Gamma}{2\delta}(\delta+U)$$

For equation (5), we assume the initial and boundary conditions as follows:

$$U(x,t)\big|_{t=0}=U(x,0)=\phi_0(x),\qquad x\in D_x=(0,1) \tag{6}$$

$$U(x,t)\big|_{x=0}=U(0,t)=\phi_1(x),\qquad t\in D_t=[0,T] \tag{7}$$

$$A(x,t)\big|_{t=0}=a_0(x),\ x\in(0,1] \tag{8}$$

$$m(x,t)\big|_{t=0}=m_0(x),\ x\in(0,1) \tag{9}$$

We will discretize equations (5) to (9) for all grid points $(x_m,t_n)\in W_k^n$ as follows

$$A(x,t)\frac{U(x,t+\Delta t)-U(x,t)}{\Delta t}=D\frac{U(x+\Delta x,t)-2U(x,t)+U(x-\Delta x,t)}{(\Delta x)^2}-$$

$$-V\frac{U(x+\Delta x,t)-U(x-\Delta x,t)}{2\Delta x}$$

For equation (1), we can discretize the initial and boundary conditions as (6)-(9) like this.

$U(x_k,0)=\varphi_0(x_k),\ x_k\in D_k,k=\overline{0,M}$

$U(1,t_j)=\varphi_0(t_j),\ t_j\in D_{j,}\ j=\overline{0,N}$

$U(1,t_j)=\varphi_2(t_j),\ \ j=\overline{0,N}$

$A(x_k,0)=a_0(x_k),\ k=\overline{0,M}$

$m(x_k,0)=m_0(x_k),k=\overline{0,M}$

$A(x,t)^{i=i}{}_x=A_k^n$

$A(x,t)=U(x,t)\big|_{x=x_k}=U_k^n$

Indicating, we can express the discretized equation as follows:

$$A_k^n \frac{U_k^{n+1} - U_k^n}{\Delta t} = D\frac{U_{k+1}^n - 2U_k^n + U_{k-1}^n}{(\Delta x)^2} - \vartheta \frac{U_{k+1}^n - U_{k-1}^n}{2 \cdot \Delta x} \qquad (10)$$

Let's approximate equation (5) using the finite difference scheme:

$$A_k^n \frac{U_k^{n+1} - U_k^n}{\Delta t} = D\frac{U_{k+1}^{n+1} - 2U_k^{n+1} + U_{k-1}^{n+1}}{(\Delta x)^2} - \vartheta \frac{U_{k+1}^{n+1} - U_{k-1}^{n+1}}{2\Delta x} \qquad (11)$$

$\sigma = 0$ (explicit scheme), $\sigma = 1$ (implicit scheme), and equations (10) and (11) can be written as follows [6]:

$$U_k^{n+1} - U_k^n = A_k^n\left(U_{k+1}^n - 2U_k^n + U_{k-1}^n\right) - A_{2,k}\left(U_{k+1}^n - U_{k-1}^n\right) = L_1 U_1^n \qquad (12)$$

$$U_k^{n+1} - U_k^n = A_k^n\left(U_{k+1}^n - 2U_k^n + U_{k-1}^n\right) - A_{2,k}\left(U_{k+1}^{n+1} - U_{k-1}^{n+1}\right) = L_1 U_k^{N+1} \qquad (13)$$

By multiplying and summing equations (12) and (13) appropriately with σ and $1-\sigma$, we get

$$U_m^{n+1} - U_m^n = \sigma U_m^{n+} + (1-\sigma)U_m^{n+1}\ ,\ 0 \le \sigma \le 1 \qquad (14)$$

When $\sigma = 0$, equation (12) is derived from (14), and when $\sigma = 1$, equation (13) is obtained. Explicit and implicit finite difference schemes have their advantages and drawbacks. In the explicit scheme, the number of operations required when transitioning from the n-th time step to the $n+1$-th time step is proportional to the number of grid points.

This is stable only in a small time domain. In contrast, the implicit scheme requires a number of operations proportional to the cube of the number of grid points n^3, and it is stable only in a wide time domain

$$N = O(M),\ N = O\left(M^3\right)$$

Let's evaluate the error of approximation. Suppose that,

$$U_m^n = U\left(x_{m,} t_n\right) + \delta U_m^n \qquad (15)$$

Here, $U\left(x_{m,} t_n\right)$ represents the exact solution of equation (5) on the grid, U_m^n is the approximate solution, and δU_m^n is the approximation error of the solution on the grid

If we consider equation (15) in light of (14) and assume that $U\left(x_{m,} t_n\right)$ is a smooth solution, we obtain:

$$\frac{\delta U_m^{n+1n} - \delta U_m^n}{\tau} = \sigma \cdot \delta U_m^{n+1} + (1-\sigma)L\delta U_m^{n+1} + \left[\frac{\partial U}{\partial t} - LU\right]_{x_m}^{x_{m+1}} + o(t, (Ax)^2) = 0$$

Since $U\left(x_m, t_n\right)$ is the exact solution

$$\left[\frac{\partial U}{\partial t}-\alpha,U\right]_{x_m}^{x_{md}}=0$$

Thus,

$$\delta U_m^n = o\left(\Delta t,(\Delta x)^2\right)$$

It follows that discrete equation (14) approximates equation (5) from the first formulation with respect to t, and from the second formulation with respect to x. If we write equation (1) in matrix form, we get :

$$A_m^n U^{n+1} = B_m^n \quad (16)$$

Equation (16) is solved using the method of sweeping. A_m^n is a tridiagonal matrix. The stability condition for the tridiagonal matrix A_m^n is satisfied :

$$b_m^n \geq a_b^n + c_m^n$$

The sweeping method distinguishes itself with high efficiency. The number of operations is proportional to the number of variables. Moreover, besides approximating the solution on the grid and ensuring stability, the exact solution accumulates from the approximate solution found on the grid.

RESULTS AND DISCUSSION

Let's investigate the asymptotic solution of equation (5). Denoting the small parameter in front of the high-order derivative as $\varepsilon = D$, we seek the solution formally in a series form as follows:

$$U(x,t,\varepsilon)=\sum_{k=0}^{\infty}\varepsilon^k U_k(x,t) \quad (17)$$

Let's take the particular sum of n terms

$$U_n(x,t,\varepsilon)=\sum_{k=0}^{n}\varepsilon^k U_k(x,t) \quad (18)$$

and examine equation (5) for small values of ε.
Let's write the expansion of (17) in equation (5) :

$$L(x,U_n(x,t,\varepsilon))=0 \;\; x\in W \in R^n,\;\; \varepsilon>0$$

$U(x,t,x,\varepsilon)$ also satisfies any boundary conditions. Suppose $U(x,t,x,\varepsilon)$ is a sequence of functions such that the following inequality holds:

$$\left|L(x,t,U_n(x,t,\varepsilon))\right| < M\cdot\varepsilon^n$$

M is independent of (x and t). If we express the specific sum of the formal asymptotic expansion (17) as (18), and if the function $U(x,t,\varepsilon)$ approximates the function $U(x,t,\varepsilon)$ for small values of ε, then the series (17) is asymptotic as $\varepsilon \to 0$ for the solution of $U(x,t,\varepsilon)$.

CONCLUSION

In laboratory experimental research, adsorption processes in the filtration of dispersed systems in a homogeneous environment have been evaluated. Graphs were obtained for pressure recovery curves in the experimental environment model under the same conditions before and after filtration of samples. Effective algorithms were developed for the numerical solution of equations considering adsorption and convective diffusion in the filtration of substances in a homogeneous environment, using numerical methods and asymptotic approaches. Here, the equation is discretized in the grid domain W_h^τ, and the new equation obtained is solved by the algorithm and the driving method. Approximation of the Tor equation with first-order accuracy with respect to time t and with second-order accuracy with respect to x has been proven, and its error has been estimated.

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HETEROGEN MAYELƏRİN SÜZÜLMƏ VƏ AXINLARINDA STRUKTUR POZULMALARIN TƏDQİQİ

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Heterogen mayelərin axınları zamanı yaranan bir sıra problemlər onların tərkibində yaranan dəyişilmələrlə əlaqədardır. Heterogen mayelərin tərkibində baş verən dəyişilmələri öyrənmək tərkibi təşkil edən mayelərin sərhəddində yaranan struktur dayanıqlığının qiymətləndirilməsində vacib faktorlardan biridir. Bu səbəbdən işdə dispers sistemlərin süzülməsində adsorbsiya prosesinin qiymətləndirilməsiylə bağlı müxtəlif eksperimentlər aparılmış və təzyiq-bərpa əyriləri qurulmuşdur. Eləcə də, məhlulların süzülməsində adsorbsiya və konvektiv diffuziya nəzərə alınaraq prosesi ifadə edən tənliklərin ədədi və asimptotik üsullarla həlli üçün alqoritm alınmışdır.

Açar sözlər: *adsorbsiya, təzyiq, konsentrasiya, diffuziya, məsaməli mühit.*

ИССЛЕДОВАНИЕ СТРУКТУРНЫХ РАЗРУШЕНИЙ ПРИ ФИЛЬТРАЦИИ И ТЕЧЕНИИ ГЕТЕРОГЕННЫХ ЖИДКОСТЕЙ

И.Дж. Мамедов, П.Т. Мусейбли

Ряд проблем, возникающих при течении гетерогенных жидкостей, связан сизменениями, происходящими в их составе. Изучение этих изменений в гетерогенных жидкостях является одним из решающих факторов в оценке структурной стабильности на границе раздела составляющих их жидкостей. В связи с этим проведены различные эксперименты, связанные с оценкой процесса адсорбции при фильтрации дисперсных систем, построены кривые восстановления давления. Дополнительно разработан алгоритм численного и асимптотического решения уравнений, описывающих процесс, с учетом адсорбции и конвективной диффузии при фильтрации растворов.

Ключевые слова: *адсорбция, давление, концентрация, диффузия, пористая среда.*