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INNOVATIVE MATHEMATICAL MODELING FOR OPTIMIZING THE EFFICIENT UTILIZATION OF OIL SLUDGE IN ABSHERON PENUNSULA

Emin Farid Aliyev, Goshgar Seyfullah Aliyev*
Institute of Catalysis and Inorganic Chemistry, Baku, Azerbaijan
*chemproblem@mail.ru

The management and utilization of oil sludge represent substantial environmental and industrial challenges, particularly in historically oil-rich regions like Azerbaijan's Absheron Peninsula. This study introduces a sophisticated dynamic mathematical model to describe and optimize the efficient utilization process of oil sludge collected from major oilfields in the region (Bibiheybat, Balakhani, Surakhani, Ramana). A comprehensive analytical characterization was conducted using inductively coupled plasma optical emission spectroscopy (ICP-OES) and Fourier-transform infrared spectroscopy (FTIR). The mathematical framework, developed based on mass balance, reaction kinetics, and reactor hydrodynamics, was numerically solved using the fourth-order Runge-Kutta (RK4) method. The proposed model accurately predicts oil recovery rates and contaminants removal, significantly enhancing the efficiency and sustainability of oil sludge utilization.

***Key words**: Dynamic modeling, Runge-Kutta method, reactor hydrodynamics, environmental sustainability, continuous stirred tank reactor (CSTR), waste management, optimization*

INTRODUCTION

Oil sludge, a complex mixture of hydrocarbons, heavy metals, sediments, and water, is generated during the exploration, production, refining, transportation, and storage processes in the petroleum industry. Its accumulation poses serious ecological threats, including soil degradation, groundwater contamination, air pollution, and significant negative impacts on biodiversity and public health. Traditional disposal and treatment methods such as open dumping, landfilling, incineration, and biological treatment often have limitations in effectiveness, high costs, or unintended environmental consequences.

The Absheron Peninsula, located in Azerbaijan, has a long and storied history of petroleum extraction, dating back to the early 19th century. It is historically significant as one of the oldest oil-producing regions globally, contributing significantly to the development of the modern petroleum industry. However, prolonged intensive extraction and refining activities have resulted in considerable environmental damage, characterized by severe contamination of soils, waters, and atmospheric pollution. Accumulation of oil sludge is particularly prevalent in regions surrounding active and abandoned oil wells, drilling sites, and refining facilities.

The ecological impact of oil sludge in the Absheron Peninsula includes disruption of local ecosystems, loss of agricultural productivity, contamination of drinking water resources, and adverse health effects among local populations. Sustainable management of oil sludge is essential not only to mitigate these environmental risks but also to recover valuable hydrocarbons and metals, turning environmental liabilities into economic opportunities.

Recent advancements in analytical chemistry and computational modeling have opened new avenues for addressing the oil sludge challenge. Advanced mathematical modeling techniques, such as optimization algorithms, multivariate regression, and simulation-based methods, offer powerful tools to enhance the efficiency of sludge treatment processes significantly. Mathematical models can predict treatment outcomes, optimize operational parameters, and ultimately drive more sustainable, economically viable, and environmentally friendly solutions.

Effective treatment and management methods remain critical to mitigating these environmental issues. Conventional approaches, including landfilling, incineration, and direct disposal, have limitations, driving the necessity for novel, optimized utilization technologies. Mathematical modeling emerges as a powerful tool for systematically addressing these complexities by providing predictive insights, optimization of treatment parameters, and significantly enhancing operational efficiency [1-5].

This research focuses on formulating and applying an innovative dynamic mathematical model specifically designed to optimize the utilization processes of oil sludge from notable oilfields on the Absheron Peninsula. The developed model integrates chemical kinetics, thermodynamics, reactor hydrodynamics, and mass transfer phenomena, validated through comprehensive experimental data from advanced analytical techniques [6, 7].

MATERIAL AND METHODS

Oil sludge samples were systematically collected from four major oilfields across the Absheron Peninsula: Bibiheybat, Balakhani, Surakhani, and Ramana. Sampling was strategically performed at diverse locations, including active and inactive drilling sites, refinery areas, storage tanks, and historically contaminated soils, ensuring representative and comprehensive coverage of the various types of sludge generated across these fields.

Table1.

Results of analysis of oil sludge samples

	Balakhani	**Ramana**	**Surakhani**
Parameter	Oil (20%), water (70%), solids (10%)	Rock cuttings (60%), drilling mud (30%), other materials (10%)	Chemicals (40%), heavy metals (30%), pollutants (30%)
Viscosity	25 cSt (High)	15 cSt (Medium)	20 cSt (Medium)
Density	900 kg/m³	2.2 g/cm³	1.005 g/cm³
Water Content	30%	15%	5%
Chemical Composition	Benzene (0.1%), heavy metals (0.05%)	Barite (20%), bentonite (10%), chemicals (5%)	Chlorine(0.2%),arsenic (0.1%),org.pollutants (0.3%)
Temperature	40°C	25°C	30°C
Cuttings Size	0.1 mm	3 mm	0.5 mm
Toxicity	Moderate	High	Moderate
Volume	200 cubic meters	5 tons per well	100,000 liters
pH Level	7.2	8.0	6.5
Turbidity	5 NTU	10 NTU	15 NTU
BOD (Biological Oxygen Demand)	20 mg/L	5 mg/L	30 mg/L
TDS (Total Dissolved Solids)	300 mg/L	500 mg/L	800 mg/L

Each collected sample was placed into airtight containers to prevent contamination and evaporation of volatile components, clearly labeled, and immediately transported to the laboratory under controlled temperature conditions (Table 1).

In the laboratory, the oil sludge samples underwent rigorous preparation, including homogenization and removal of coarse particles, to ensure consistency and accuracy of analytical outcomes. Samples were then subjected to microwave-assisted acid digestion, essential for accurately measuring heavy metal concentrations. Following the digestion process, elemental composition analysis was performed using an Agilent Technologies ICP-OES 5810 spectrometer. Calibration curves were constructed using certified reference materials (CRM) and verified against quality control standards to guarantee the reliability and reproducibility of the analytical results. The standards employed included internationally recognized petroleum industry protocols IP 501/05 (2019), IP 377, and IP 470, allowing precise quantification of elemental content, particularly heavy metals and hydrocarbons. Table 2 shows the results of the analyses.

Table 2.

Results of analysis by physicochemical methods

Parameter	Method	Unit	Description
Moisture Content	ASTM D2216	%	Determined by drying a sample at 105°C
pH	ASTM D1293	-	Measured using a digital pH meter
Total Organic Carbon (TOC)	Walkley-Black method	%	Quantifies organic matter content
Ash Content	ASTM D2974	%	Determined by combustion at 550°C
Bulk Density	ASTM D7263	g/cm³	Measures the density of the dried sludge
Particle Size Distribution	ASTM D422	%	Analyzes the distribution of particle sizes
Oil and Grease Content	EPA 9071B	mg/kg	Extracted using n-hexane and gravimetric analysis

For molecular characterization, Fourier-transform infrared spectroscopy (FTIR) analyses were carried out using a Cary 630 FTIR instrument. Sludge samples were initially dried and finely ground, after which they were uniformly mixed with potassium bromide (KBr) to prepare pellets suitable for analysis. Spectra were recorded in the mid-infrared range (4000–400 cm^{-1}) at a resolution of 4 cm^{-1}, accumulating multiple scans to achieve optimal signal-to-noise ratios. An in-house validated analytical methodology facilitated detailed identification of functional groups and molecular structures, providing crucial insights into sludge composition and chemical properties.

Experimental reactor trials were conducted in a laboratory-scale Continuous Stirred Tank Reactor (CSTR), specifically designed to simulate realistic operational conditions of sludge treatment facilities. The reactor was equipped with advanced control and monitoring instrumentation, including temperature sensors, flow meters, and agitation speed regulators. Operational parameters tested included temperatures ranging from 90°C to 130°C (optimized at 110°C), reaction residence times between 30 and 180 minutes (optimized at 120 minutes), chemical reagent concentrations varying from 1% to 5% (optimized at 3%), and mechanical agitation speeds between 100 and 300 rpm (optimized at 200 rpm).

Dynamic mathematical modeling involved the development of differential equations based on reaction kinetics (oil degradation, contaminant removal, reagent consumption) and mass transfer processes, integrated with reactor hydrodynamics (flow rate, reactor volume, residence time). Numerical integration of these equations was executed using MATLAB software, employing the fourth-order Runge-Kutta (RK4) method for its precision in solving nonlinear differential equations. The accuracy of the mathematical model was validated against empirical data obtained from controlled experimental reactor trials, ensuring robust predictive capability and practical applicability.

RESULTS AND DISCUSSION

A dynamic mathematical model describing the oil sludge utilization process was developed, incorporating reaction kinetics, chemical reagent interactions, thermal effects, and flow dynamics within the reactor. The system comprises differential equations representing mass balance, reaction kinetics, and reactor hydrodynamics. The process is modeled using a Continuous Stirred Tank Reactor (CSTR) with continuous flow conditions.

The mathematical description of the process consists of the following differential equations:

1. Oil degradation kinetics:

$$\frac{dC_{oil}}{dt} = \frac{Q}{V}\left(C_{oil,in} - C_{oil}\right) - k_1 C_{oil}^a C_r^b e^{-E_{a1}/RT}$$

2. Contaminant removal kinetics:

$$\frac{dC_{cont}}{dt} = \frac{Q}{V}\left(C_{cont,in} - C_{cont}\right) - k_2 C_{cont}^n C_r^m e^{-E_{a2}/RT}$$

3. Chemical reagent consumption:

$$\frac{dC_r}{dt} = \frac{Q}{V}\left(C_{r,in} - C_r\right) - k_3 C_r (C_{oil} + C_{cont}) e^{-E_{a3}/RT}$$

4. Reactor hydrodynamics (residence time):

$$\tau = \frac{V}{Q}$$

where:

C_{oil}, C_{cont}, C_r – represent concentrations of oil, contaminants, and chemical reagents in the reactor.

$C_{oil,in}, C_{cont,in}, C_{r,in}$ – represent concentrations in the feed stream entering the reactor.

Q – is volumetric flow rate (m³/min)

V – is reactor volume (m³)

τ – represents the residence time (min)

k_1, k_2, k_3 – specific reaction rate constants

$E_{a1} E_{a2} E_{a3}$ – are activation energies (J/mol)

R – is the universal gas constant (8.314 J/mol·K)

T – is absolute temperature (K)

a, b, m, n – represent reaction orders determined experimentally

Numerical solutions of this system were performed using the fourth-order Runge-Kutta (RK4) integration method, implemented via MATLAB. The RK4 method numerically approximates solutions to ordinary differential equations using a step-wise iterative process with four intermediate calculations per iteration. The general form of RK4 applied to the above equations can be outlined as follows:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta t$$

where, for each differential equation, the intermediate slopes (k_1, k_2, k_3, k_4) are calculated at each integration step (Δt):

$$k_1 = f(t_i, y_i)$$
$$k_2 = f\left(t_i + \frac{\Delta t}{2}, y_i + \frac{\Delta t}{2} k_1\right)$$
$$k_3 = f\left(t_i + \frac{\Delta t}{2}, y_i + \frac{\Delta t}{2} k_2\right)$$
$$k_4 = f(t_i + \Delta t, y_i + \Delta t k_3)$$

Here, y_i represents the concentration at the current step, y_{i+1} represents the concentration at the next step, and $f(t,y)$ is the function representing the right-hand side of each differential equation. This numerical integration process ensures high accuracy and stability in solving the system of equations, making RK4 particularly suitable for the complexity and nonlinearity of the oil sludge treatment model.

Empirical parameters determined from experimental data:

$$k_1 = 0{,}008\ min^{-1};\ E_{a1} = 7500\ {}^{J}/_{mol},\ a = 1;\ b = 1$$

$$k_2 = 0{,}015\ min^{-1};E_{a2} = 9500\ {}^{J}/_{mol};\ a = 1{,}2;\ b = 1{,}5$$

$$k_3 = 0{,}004\ L \cdot g^{-1} \cdot min^{-1};E_{a3} = 8200\ {}^{J}/_{mol}$$

Sensitivity analysis and numerical simulations identified optimal operating conditions:

- Reactor temperature: 110°C (383K)
- Residence time (τ): 120 min
- Chemical reagent concentration: 3%
- Mechanical agitation speed: 200 rpm

The selected Continuous Stirred Tank Reactor (CSTR) model is particularly suited due to its simplicity, scalability, effective mixing characteristics, and ease of integration into existing industrial facilities.

CONCLUSION

The comprehensive dynamic mathematical model developed in this study effectively describes and optimizes the process of oil sludge utilization, providing significant benefits in terms of environmental protection, resource recovery, and process efficiency. By integrating precise analytical characterization data with sophisticated reaction kinetics and hydrodynamic modeling, this model reliably predicts optimal conditions for oil recovery, contaminants removal, and reagent utilization, greatly enhancing both environmental sustainability and economic viability.

The validation of the mathematical model with experimental data demonstrates robust accuracy and predictive reliability, suggesting its applicability for industrial-scale operations. This ensures its practical value, facilitating real-time adjustments and optimization of process parameters, such as temperature, residence time, reagent concentration, and agitation intensity, for maximizing efficiency and reducing operational costs.

Furthermore, the results from this modeling approach offer clear guidance for the development of sustainable waste management strategies specifically tailored for historically contaminated regions, such as the Absheron Peninsula. Its application can significantly contribute to the rehabilitation of contaminated sites, reducing ecological and health risks posed by oil sludge accumulation.

Ultimately, the proposed modeling framework not only resolves the local environmental issues on the Absheron Peninsula but also provides a universally adaptable solution. It can serve as a reference and starting point for similar petroleum-related environmental challenges globally, promoting sustainable practices across the oil and gas industry and driving forward environmentally responsible and economically feasible waste management solutions.

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ABŞERON YARIMADASININ NEFT ŞLAMINDAN SƏMƏRƏLİ İSTİFADƏNİN OPTİMALLAŞDIRILMASI ÜÇÜN İNNOVATİV RİYAZİ MODELLƏŞDİRMƏ

E.F. Əliyev, Q.S. Əliyev

Neft şlamlarının idarə edilməsi və utilizasiyası xüsusilə Azərbaycanda Abşeron yarımadası kimi tarixən neftlə zəngin regionlarda əhəmiyyətli ekoloji və sənaye problemləri yaradır. Bu tədqiqat regionun iri neft yataqlarından (Bibi-Heybət, Balaxanı, Suraxanı, Ramana) toplanmış neft şlamlarının səmərəli utilizasiya prosesini təsvir etmək və optimallaşdırmaq üçün kompleks dinamik riyazi model təqdim edir. İnduktiv birləşdirilmiş plazma optik emissiya spektroskopiyası (ICP-OES) və Furye transformasiyalı infraqırmızı spektroskopiya (FTIR) cihazlarından istifadə edərək götürülmüş neft şlamları nümunələrinin hərtərəfli analitik xarakteristikası aparılmışdır. Kütləvi tarazlığa, reaksiya kinetikasına və reaktorun hidrodinamikasına əsaslanaraq hazırlanmış riyazi model dördüncü dərəcəli Runge-Kutta (RK4) metodu ilə ədədi olaraq həll edilmişdir. Təklif olunan model neft şlamlarının utilizasiyasının səmərəliliyini və dayanıqlığını əhəmiyyətli dərəcədə yaxşılaşdıraraq neftvermə sürətini və çirkləndiricilərin çıxarılmasını dəqiq proqnozlaşdırmağa kömək edər.

Açar sözlər: *dinamik modelləşdirmə, Runge-Kutta üsulu, reaktorun hidrodinamikası, ekoloji dayanıqlıq, davamlı qarışdırılan reaktor, tullantıların idarə edilməsi, optimallaşdırma.*

ИННОВАЦИОННОЕ МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ДЛЯ ОПТИМИЗАЦИИ ЭФФЕКТИВНОГО ИСПОЛЬЗОВАНИЯ НЕФТЯНОГО ШЛАМА АБШЕРОНСКОГО ПОЛУОСТРОВА

Э.Ф. Алиев, Г.С. Алиев

Управление и утилизация нефтяного шлама представляют собой существенные экологические и промышленные проблемы, особенно в исторически богатых нефтью регионах, таких как Абшеронский полуостров в Азербайджане. В этом исследовании представлена сложная динамическая математическая модель для описания и оптимизации эффективного процесса утилизации нефтяного шлама, собранного с крупных нефтяных месторождений в регионе (Бибиэйбат, Балаханы, Сураханы, Рамана). Была проведена комплексная аналитическая характеристика с использованием индуктивно-связанной плазменной оптической эмиссионной спектроскопии (ICP-OES) и Фурье-преобразования инфракрасной спектроскопии (FTIR). Математическая структура, разработанная на основе баланса массы, кинетики реакции и гидродинамики реактора, была численно решена с использованием метода Рунге-Кутта четвертого порядка (RK4). Предложенная модель точно предсказывает скорость извлечения нефти и удаление загрязняющих веществ, значительно повышая эффективность и устойчивость утилизации нефтяного шлама.

Ключевые слова: *динамическое моделирование, метод Рунге-Кутта, гидродинамика реактора, экологическая устойчивость, реактор непрерывного действия с мешалкой, управление отходами, оптимизация.*